Introducing an inter-model helps decreasing the degrees of freedom in EIT inverse problem

Rongqing Chen[1,2], Alberto Battistel[1], Sabine Krüger-Ziolek[1], Stefan J. Rupitsch[2], Knut Möller[1]

[1]Institute for Technical Medicine (ITeM), Hochschule Furtwangen, Villingen-Schwenningen, Germany, chr@hs-furtwangen.de

[2]Faculty of Engineering, University of Freiburg, Freiburg, Germany

Abstract: EIT reconstruction is characterized with large degrees of freedom. In this contribution, we introduced an 'inter-model', which is the basic function subset derived from Discrete Cosine Transformation (DCT), to cluster the elements of the FEM. This decreases its degrees of freedom. The reconstruction shows more precision and less artefacts.

1 Introduction

Electrical Impedance Tomography (EIT) is an imaging modality mainly used to generate two-dimensional cross-sectional images representing impedance change in the thorax. Despite all the benefits, the inverse problem of EIT is ill-posed and characterized with large degrees of freedom. Gong et al. proposed an algorithm to decrease its degrees of freedom. It uses the spectral graph wavelets to downsample the nodes in the Finite Element Model (FEM) used in the reconstruction [1]. However, the downsampling is based on a reconstruction generated from less iterative steps, but this is still characterized with the original degrees of freedom. In this contribution, we introduced a basic function subset derived from Discrete Cosine Transformation (DCT) as an 'inter-model', which modifies the mapping of the Jacobian matrix $\mathbf{J}$. Hence, the degrees of freedom in the inverse problem are decreased. We conducted numerical simulations to generate the voltage data for the reconstruction purpose. Figures of merit are used to compare the EIT images from two different EIT algorithms, which are the Gauss-Newton linear one-step solver (GN approach) and our DCT approach.

2 Methods

The estimation of the changes in conductivity $\hat{\mathbf{x}}$ within the thorax from a set of changes in boundary voltages $\mathbf{y}$ is described in EIT as a simplified linearized form:

$$\hat{\mathbf{x}} = (\mathbf{J}^T\mathbf{J} + \lambda^2\mathbf{R})^{-1}\mathbf{J}^T\mathbf{y} = \mathbf{B}\mathbf{y}\,. \quad (1)$$

An 'inter-model' $\mathbf{D}$, which is a subset of basic cosine functions at varying frequencies derived from DCT, is introduced as

$$D(p,q)_{m,n} = \alpha_p\alpha_q \cos\frac{(2m+1)p\pi}{2M}\cos\frac{(2n+1)q\pi}{2N}\,, \quad (2)$$

where p and q are the frequencies of the cosine functions, which were chosen as 15 for both axis here. The desired image size is $M \times N$, and (m,n) is the position of a pixel.

The columns of the basic function subset are calculated from $\mathbf{D}(p,q)$ as $\mathbf{K}_j = T(\mathbf{D}(p,q))$. T is a map to assign every pixel in $\mathbf{D}(p,q)$ to the FEM element, which covers the pixel. and j is the column index of $\mathbf{K}$. It is calculated as $j = q \cdot \sqrt{N_{DCT}} + p + 1$, e.g., $\mathbf{K}_3 = T(C(1,1))$.

The Jacobian matrix $\mathbf{J}$ is modified by the subset $\mathbf{K}$ as $\mathbf{J}_{DCT} = \mathbf{JK}$. Hence, $\mathbf{J}_{DCT}$ maps the voltage variations to the DCT coefficients change. $\mathbf{J}_{DCT}$ contains only $n_{meas} \times n_{DCT}$ elements which are far less than the number of FEM elements. Then, the solution of the inverse problem is represented by the change of DCT coefficients $\hat{\mathbf{x}}_{DCT}$. The change of the DCT coefficients $\hat{\mathbf{x}}_{DCT}$ is used to restore the EIT image $\mathbf{H}$ through inverse DCT calculation.

In this contribution, we chose the Tikhonov regularization as $\mathbf{R} = \mathbf{I}$ for all algorithms. The optimal λ is chosen when the noise figure (NF) reaches 0.5. The simulations were conducted with MATLAB R2019a (Mathworks, MA, USA) using the EIDORS toolbox [2]. 1% of the white noise was added to the voltages from the simulation.

3 Results

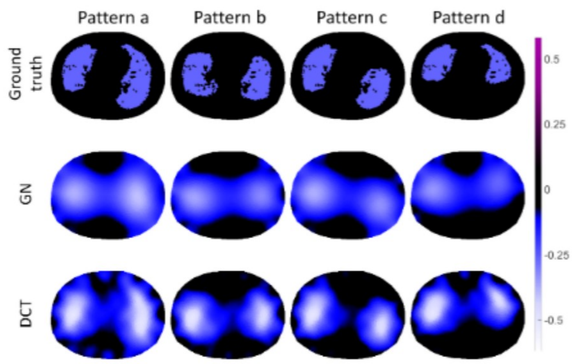

Figure 1: Ground truth and the EIT reconstructions. First row: different patterns of conductivity change for simulation (ground truth); second row: reconstructions using the GN approach; third row: reconstructions using DCT approach.

Table 1: The figures of merit of the reconstructions using GN approach and DCT approach.

Figures of merit	Dislocation		Shape deformation		Ring effect		Amplitude response	
Algorithm	GN	DCT	GN	DCT	GN	DCT	GN	DCT
Pattern a	0.092	0.054	1.850	1.063	1.224	0.485	2.283	1.413
Pattern b	0.074	0.052	1.942	0.954	1.216	0.495	1.998	1.310
Pattern c	0.098	0.062	1.964	0.978	1.264	0.455	2.133	1.330
Pattern d	0.101	0.075	2.265	1.200	1.262	0.585	2.264	1.344

4 Conclusions

With the introduction of the 'inter-model' into the EIT reconstruction, the degrees of freedom in the inverse problem are reduced. According to the EIT images and the corresponding quantitative analysis, the DCT approach leads to the better results with more precision.

References

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